

Lecture 2: Statics of Particles

September 13, 2016 1:01 PM

2.2 Resultant of two forces

A **force** is an action of one body on another

- it has a **point of application**, **magnitude**, and **direction**

Multiple forces acting on a particle have the **same** point of application, so generally we only deal with forces defined with their magnitude and direction (remember: they're vectors)

Remember:

- magnitude is measured in newtons (N)/kilonewtons (kN)
- direction is defined as the **line of action** (infinite straight line), **sense** of the force, and angle is forms with a fixed axis

2 forces acting on a particle can be replaced by a single force, **R**, the **resultant** (see in Lecture 1 note: Parallelogram Law for the Addition of 2 Forces)

2.3 Vectors

Vectors are mathematical expressions possessing magnitude and direction.

- forces, displacements, velocity, accelerations, and momenta are all vectors

Vectors can be **fixed** or **bound** if they represent a force acting on a particle at a well-defined point, whereas a **free vector** can be moved anywhere in space.

Sliding vectors can be moved (or slid) along their lines of action

- 2 vectors of **same magnitude** and **same direction** are said to be **equal**
- 2 vectors of **same magnitude** but **opposite direction** are **negative** vectors

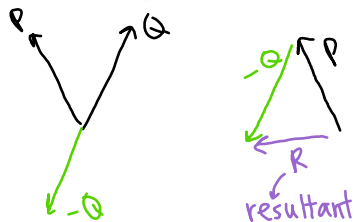
2.5 Addition of forces

Vector addition is **commutative** meaning they can be added together in any order.

- attaching vectors tip-tail and turning them into a triangle allows us to use trigonometry to determine the magnitude of the resultant
- vectors can't be added like scalar quantities, because direction must be taken into account

Vector subtraction is defined as the addition of the corresponding negative vector:

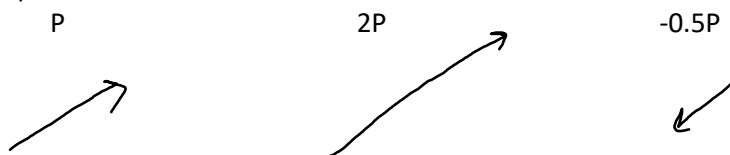
$$P - Q = P + (-Q)$$



The sum of multiple vectors involves summing the first two, then adding the third to it, then adding the fourth to the sum of that, then adding the 5th to the sum of *that*, and so on. The order doesn't matter, meaning the addition is **associative**.

A vector can be multiplied by any scalar quality (that is a real number).

For example:



2.5 Resultant of several concurrent forces

Coplanar forces are contained on the same plane

Concurrent forces are those passing through the same point

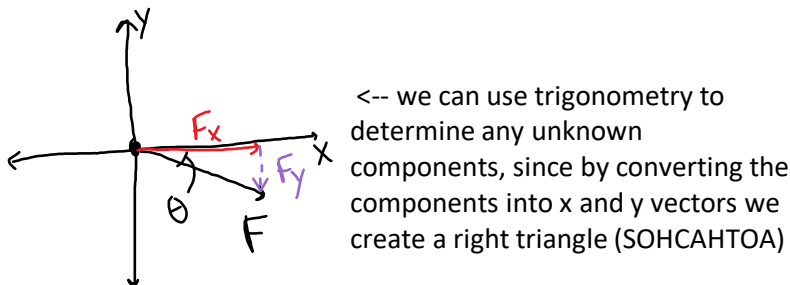
The trigonometric method for vector addition mentioned above can be expanded

for more than 2 vectors. This is the **polygon rule**, and vectors are attached tip-to-tail to find the result.

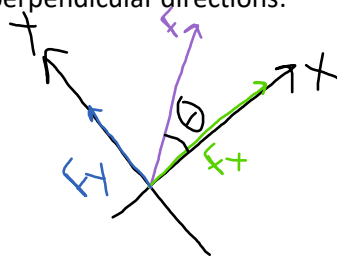
2.6 Resolution of a force into components

*This is the opposite of adding forces. Instead, we break them into 2 parts that **equal** the original force. These parts are called **components** of the original force **F**. The process of determining components is called **resolving the force into components**.

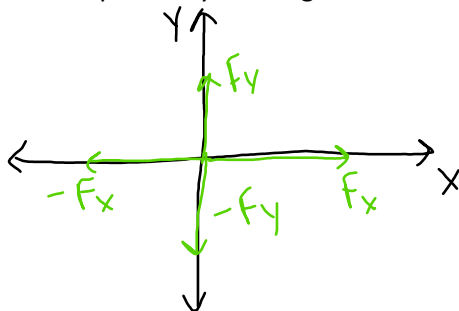
For a force **F**, there are an infinite number of possible components. Defining a perpendicular axis determines the horizontal and vertical components of the force, called **rectangular components**. This is important for practical applications. If we know one of the components, say F_x , and F , we can determine the vertical component, or F_y .



Typically x and y axes are drawn horizontal and vertical, they can technically be in any two perpendicular directions:



F_x is positive with positive x and negative in the opposite sense.
 F_y is positive with positive y and negative in the opposite sense.



If **F** is the magnitude of the force and θ is the angle between **F** and the x-axis, the scalar components of **F** are:

$$F_x = F \cos \theta \quad \text{and} \\ F_y = F \sin \theta$$



The resultant of 2 forces can be determined graphically or trigonometrically, but with 3 or more forces trigonometry cannot be applied.

Instead, we can determine the resultant by **resolving each force into 2 rectangular components**:

$$R_x = P_x + Q_x + S_x \quad \text{and} \quad R_y = P_y + Q_y + S_y$$

or

$$R_x = P_x + Q_x + S_x \quad \text{and} \quad R_y = P_y + Q_y + S_y$$

or

$$R_x = \Sigma F_x \quad \text{and} \quad R_y = \Sigma F_y$$